

Funkcje zespolone

Zera funkcji

Zadanie 1. Znaleźć zera. Wyznaczyć ich rzędy.

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| (1) $f(z) = 1 - e^z$; | (4) $f(z) = z^4 + 4z^2$; |
| (2) $f(z) = \frac{z^8}{z - \sin z}$; | (5) $f(z) = z^2 \sin z$; |
| (3) $f(z) = (z^2 + 1)^3 \operatorname{sh} z$; | (6) $f(z) = 1 + \operatorname{ch} z$. |

Punkty osobliwe odosobnione

Zadanie 2. Wyznaczyć punkty osobliwe i określić rodzaj osobliwości.

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| (1) $f(z) = \frac{e^z - 1}{z}$; | (5) $f(z) = \frac{1}{1 - \sin z}$; |
| (2) $f(z) = \frac{1}{z^3}$; | (6) $f(z) = \frac{1 + \cos z}{z - \pi}$; |
| (3) $f(z) = \frac{\sin z}{z^3 + z^2 - z - 1}$; | (7) $f(z) = \frac{z^2 - 3z + 2}{z^2 - 2z + 1}$; |
| (4) $f(z) = e^{z^2}$; | (8) $f(z) = \frac{\sin z}{z^2}$. |

Residua

Zadanie 3. Znaleźć residua w punktach osobliwych.

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| (1) $f(z) = \frac{\sin z^2}{z^3 - \frac{\pi}{4} z^2}$; | (4) $f(z) = z^3 \sin \frac{1}{z^2}$; |
| (2) $f(z) = \frac{e^z}{(z+1)^3(z-2)}$; | (5) $f(z) = \frac{\operatorname{ch} z}{(z^2+1)(z-3)}$; |
| (3) $f(z) = \frac{1}{z^4+1}$; | (6) $f(z) = \frac{e^z}{z^3(z-1)}$. |

Zadanie 4. Obliczyć całki.

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| (1) $\int_{ z =4} \frac{e^z - 1}{z^2 + z} dz$; | (4) $\int_{ z =2} \frac{e^z}{z^3(z+1)} dz$; |
| (2) $\int_{ z =2} \operatorname{tg} z dz$; | (5) $\int_{ z =1/2} z^2 \sin \frac{1}{z} dz$; |
| (3) $\int_{ z-i =3/2} \frac{e^{-z^2}}{z^2+1} dz$; | (6) $\int_{ z =\sqrt{3}} \frac{\sin \pi z}{z^2 - z} dz$; |

Zadanie 5. Obliczyć całki typu $\int_{-\infty}^{\infty} R(x) dx$, $R(x) := \frac{P_m(x)}{Q_n(x)}$.

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| (1) $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)^2}$, $(a > 0)$; | (3) $\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$, $(a > 0, b > 0)$; |
| (2) $\int_0^{\infty} \frac{x^2+1}{(x^4+1)} dx$; | (4) $\int_0^{\infty} \frac{dx}{(x^2+1)^3}$, $(a > 0)$. |

Zadanie 6. Obliczyć całki typu $\int_0^{\infty} R(x) \cos \lambda x dx$, $\int_0^{\infty} R(x) \sin \lambda x dx$.

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| (1) $\int_0^{\infty} \frac{x \sin ax dx}{x^2+b^2}$, $(a > 0, b > 0)$; | (3) $\int_0^{\infty} \frac{x \sin x dx}{x^2+4x+20}$; |
| (2) $\int_0^{\infty} \frac{x \cos x dx}{x^2-2x+10}$; | (4) $\int_0^{\infty} \frac{\cos x dx}{x^2+9}$. |

Zadanie 7. Obliczyć całki typu $\int_0^{2\pi} R(\cos x, \sin x) dx$.

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| (1) $\int_0^{2\pi} \frac{dx}{(a+b \cos x)^2}$, $(a > b > 0)$; | (3) $\int_0^{2\pi} \frac{\cos 2x dx}{1-2p \cos x + p^2}$, $(p > 1)$; |
| (2) $\int_0^{2\pi} \frac{dx}{1-2p \cos x + p^2}$, $(0 < p < 1)$; | (4) $\int_0^{2\pi} \frac{dx}{a + \cos x}$, $(a > 1)$. |

Zadanie 8* Obliczyć całki.

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| (1) $\int_0^{\infty} \frac{\sin ax dx}{x(x^2+b^2)^2}$, $(a > 0, b > 0)$; | (3) $\int_0^{\infty} \sin x^2 dx$; |
| (2) $\int_0^{\infty} \frac{\sin x dx}{x}$; | (4) $\int_{-\infty}^{+\infty} \frac{e^{ax} dx}{1+e^x}$, $(0 < a < 1)$. |