ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2009, Suppl. 2, pp. S001–S008.

published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2009, Vol. 15, No. 1.

Integral Approximation of the Characteristic Function of an Interval and the Jackson Inequality in $C(\mathbb{T})$

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Received February 2, 2009

Abstract—An application of the results about the integral approximation of the characteristic function of an interval by the subspace $\mathcal{T}_{n-1}$ of trigonometric polynomials of order at most $n-1$, which were obtained by the authors earlier, to the investigation of the Jackson inequality between the best uniform approximation of a continuous periodic function by the subspace $\mathcal{T}_{n-1}$ and its modulus of continuity of the second order is presented. A respective method of the uniform approximation of continuous periodic functions by trigonometric polynomials is constructed.
Keywords: integral approximation of a function by polynomials, the Jackson inequality.

INTRODUCTION

Let $\mathbb{T} = [-\pi, \pi) = \mathbb{R}/(2\pi\mathbb{Z})$ be the period (the one-dimensional torus); let $L = L(\mathbb{T})$ be the space of 2π-periodic Lebesgue integrable functions $f\colon \mathbb{T} \to \mathbb{R}$ with the norm $\|f\|_L = \int_{-\pi}^{\pi} |f(t)|\,dt$; let $L_\infty = L_\infty(\mathbb{T})$ be the space of 2π-periodic functions $f: \mathbb{T} \to \mathbb{R}$ measurable and essentially bounded on $\mathbb{T}$ with the norm $\|f\|_{L_\infty} = \operatorname{ess\,sup}\{|f(t)| : t \in \mathbb{T}\}$; let $C = C(\mathbb{T})$ be the space of continuous 2π-periodic functions $f: \mathbb{T} \to \mathbb{R}$ with the norm $\|f\|_C = \max\{|f(t)| : t \in \mathbb{T}\}$. In addition to the spaces of periodic functions listed, we will also use the space $L(\mathbb{R})$ of Lebesgue integrable functions $f\colon \mathbb{R} \to \mathbb{R}$ with the norm $\|f\|_{L(\mathbb{R})} = \int_{\mathbb{R}} |f(t)|\,dt$.

We denote by $\chi_{(-h,h)}$ the characteristic function of the interval $(-h,h)$; i. e.,

$$\chi_{(-h,h)}(t) = \begin{cases} 1, & t \in (-h,h), \\ 0, & t \notin (-h,h). \end{cases}$$

Below, we use the known method of periodization (see [7, Ch. 7, Sect. 2, formula (2.1)]), which sets up a correspondence between a function $f \in L(\mathbb{R})$ and a 2π-periodic function F from $L(\mathbb{T})$ by the following formula:

$$F(t) = \sum_{j\in\mathbb{Z}} f(t + 2\pi j). \tag{0.1}$$

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For example, we set up a correspondence between the function $\chi_{(-h,h)}$ and the 2π-periodic function

$$\chi_h(t) = \sum_{j\in\mathbb{Z}} \chi_{(-h,h)}(t + 2\pi j).$$

In addition to $\chi_{(-h,h)}$ and χ_h, we consider the functions

$$\mathcal{X}_{(-h,h)}(t) = \frac{1}{2h}\chi_{(-h,h)}(t), \qquad \mathcal{X}_h(t) = \frac{1}{2h}\chi_h(t).$$

For any $h > 0$, we have

$$\|\mathcal{X}_{(-h,h)}\|_{L(\mathbb{R})} = 1, \qquad \|\mathcal{X}_h\|_{L(\mathbb{T})} = 1.$$

The first equality is evident. As for the second one, see the proof of Theorem 2.4 from [7, Ch. 7, Sect. 2]. The Fourier series of the function $\mathcal{X}_h$ has the form

$$\mathcal{X}_h(t) = \frac{1}{2\pi}\sum_{k\in\mathbb{Z}} \frac{\sin kh}{kh} e^{ikt} = \frac{1}{\pi}\left(\frac{1}{2} + \sum_{k=1}^{\infty} \frac{\sin kh}{kh}\cos kt\right); \tag{0.2}$$

we consider the relation $(\sin kh)/(kh)$ to be equal 1 for $k = 0$.

This paper continues the investigations by the authors [2, 3] devoted to the integral approximation of the function χ_h by the subspace $\mathcal{T}_{n-1}$ of the real-valued trigonometric polynomials

$$g(t) = \sum_{|k|\le n-1} c_k e^{ikt}, \quad c_k \in \mathbb{C}, \quad c_{-k} = \overline{c}_k,$$

of order at most $n - 1$.

The quantity

$$E_{n-1}(f)_L = \min\{\|f - g\|_L : \ g \in \mathcal{T}_{n-1}\}$$

is called the value of the best integral approximation of a function $f \in L$ by the subspace $\mathcal{T}_{n-1}$.

A brief history of the integral approximation of specific functions by polynomials is presented in the authors' papers [2, 3]. The value of the best integral approximation of the function $\mathcal{X}_h$ by the subspace $\mathcal{T}_{n-1}$ is found in [2] for specific h; this value is found in [3] for all $h \in (0, \pi)$. The following assertions are proved in [2, Theorem 1.3.1]:

$$E_{n-1}(\mathcal{X}_h)_L = 1 \quad \text{for} \quad 0 < h \le \frac{\pi}{2n},$$

$$E_{n-1}(\mathcal{X}_h)_L \le \frac{\pi}{2nh} < 1 \quad \text{for} \quad \frac{\pi}{2n} < h \le \pi; \tag{0.3}$$

in addition,

$$E_{n-1}(\mathcal{X}_{h_j})_L = \frac{\pi}{2nh_j}, \quad \text{where} \quad h_j = \frac{(2j-1)\pi}{2n}, \quad j = 1, 2, \ldots, n. \tag{0.4}$$

In [3, Sect. 5], we show that the sequence $\mathcal{E}_{n-1} := E_{n-1}\left(\mathcal{X}_{\pi/n}\right)_L$ $(n = 2, 3, \ldots)$ tends to the number $1 - 2v_1/\pi$ monotonically increasing, where v_1 is the unique root of the equation

$$\sec v - \tan v = v/\pi \quad \text{on the segment} \quad [0, \pi/2];$$

i.e.,

$$\mathcal{E}_1 < \mathcal{E}_2 < \ldots < \lim_{n\to\infty} \mathcal{E}_n = 1 - 2v_1/\pi = 0.3817350529\ldots.$$

It is not hard to verify that the case $h > \pi$ is reduced to the case $h \in (0, \pi]$; namely,

$$E_{n-1}(\mathcal{X}_h)_L = 0 \quad \text{for} \quad h/\pi \in \mathbb{N}, \tag{0.5}$$

$$E_{n-1}(\mathcal{X}_h)_L = \left\{\frac{h}{\pi}\right\} \frac{\pi}{h} E_{n-1}(\mathcal{X}_{\{h/\pi\}\pi})_L \quad \text{for} \quad h > 0, \quad h/\pi \notin \mathbb{N}; \tag{0.6}$$

here, $\{a\}$ denotes the fractional part of the number a. Assertion (0.5) is evident. The simplest way to justify (0.6) is to use the equality

$$E_{n-1}(\mathcal{X}_h)_L = E_{n-1}(\widetilde{\mathcal{X}}_h)_L, \quad h > 0, \tag{0.7}$$

where $\widetilde{\mathcal{X}}_h$ denotes the 2π-periodic function obtained from the function

$$\mathcal{X}_{(0,2h)}(t) = \begin{cases} \dfrac{1}{2h}, & t \in (0, 2h), \\ 0, & t \notin (0, 2h) \end{cases}$$

with the help of (0.1). In its turn, assertion (0.7) follows from the invariance of the subspace $\mathcal{T}_{n-1}$ with respect to any shift of the variable.

It follows from (0.5) and (0.6) that the restriction $h \le \pi$ in (0.3) can be lifted; i.e.,

$$E_{n-1}(\mathcal{X}_h)_L \le \frac{\pi}{2nh} < 1 \quad \text{for} \quad h > \frac{\pi}{2n}. \tag{0.8}$$

It follows from (0.4) and (0.6) that the first inequality in (0.8) turns into an equality for h coinciding with an arbitrary positive zero of the function $\cos nt$; i.e.,

$$E_{n-1}(\mathcal{X}_{h_j})_L = \frac{\pi}{2nh_j}, \quad \text{where} \quad h_j = \frac{(2j-1)\pi}{2n}, \quad j \in \mathbb{N}.$$

In this paper, we show how a sharp result about the approximation of the characteristic function of an interval in $L(\mathbb{T})$ can be applied to the investigation of the Jackson problem about estimating the best uniform approximation of a continuous periodic function in terms of the second finite difference of the function.

1. SOME NOTATION AND AUXILIARY STATEMENTS

We denote by $*$ the operation of the convolution of functions f and g, which are given on the real line (see [7, Ch. 1, Sect. 1]),

$$f*g(x) = \int_{\mathbb{R}} f(x-t)g(t)\,dt.$$

We denote by $\odot$ the operation of the convolution of functions f and g, which are given on the period (see [6, Ch. 1, Sect. 1.5, Subsect. 4], [9, Ch. 3, Sect. 3.1]),

$$f\odot g(x) = \int_{\mathbb{T}} f(x-t)g(t)\,dt. \tag{1.1}$$

Let us consider the Steklov operator $S_h:\ L \to C$ (see [1, Ch. 3, Sect. 67])

$$S_h f(x) = \frac{1}{2h}\int\limits_{x-h}^{x+h} f(t)\,dt = \frac{1}{2h}\int\limits_{-h}^{h} f(x+t)\,dt = \frac{1}{2h}\int\limits_{-h}^{h} f(x-t)\,dt, \quad h>0.$$

A function $f \in L$ can be considered as a function given on the real line. Taking into account this fact as well as the reasoning used in the proof of Theorem 2.4 from [7, Ch. 7, Sect. 2], it is not hard to show that the following relations are valid:

$$S_h f(x) = f{*}\mathcal{X}_{(-h,h)}(x) = f{\odot}\mathcal{X}_h(x) \quad \text{for any} \quad h>0. \tag{1.2}$$

We denote by $\mathcal{T}_{n-1}^{\perp}$ the set of functions $\varphi \in L_\infty$ orthogonal to the subspace $\mathcal{T}_{n-1}$, i.e., the set of functions $\varphi \in L_\infty$ such that $\int\limits_{-\pi}^{\pi} \varphi(t)\,g(t)\,dt = 0$ for all $g \in \mathcal{T}_{n-1}$.

Let $\varphi \in \mathcal{T}_{n-1}^{\perp}$ and $g \in \mathcal{T}_{n-1}$. By (1.2) and an inequality for convolutions (see [6, Ch. 1, Sect. 1.5, Subsect. 4, Proposition 1.5.5]), we have

$$|S_h\varphi(x)| = |\varphi \odot (\mathcal{X}_h - g)(x)| \le \inf_{g\in\mathcal{T}_{n-1}} \|\mathcal{X}_h - g\|_L \|\varphi\|_{L_\infty} = E_{n-1}(\mathcal{X}_h)_L \|\varphi\|_{L_\infty}.$$

Consequently, the following inequality holds for any function $\varphi \in \mathcal{T}_{n-1}^{\perp}$:

$$|S_h\varphi(x)| \le E_{n-1}(\mathcal{X}_h)_L \|\varphi\|_{L_\infty}. \tag{1.3}$$

Let us show that the inequality is sharp. We denote by $g_h \in \mathcal{T}_{n-1}$ the polynomial of the best integral approximation of the function $\mathcal{X}_h$. Then, by Markov's criterion (see, for example, [3, Theorem 2]), we have, firstly,

$$\varphi_h(t) := \operatorname{sign}\big(\mathcal{X}_h(t) - g_h(t)\big) \in \mathcal{T}_{n-1}^{\perp}$$

and, secondly,

$$|S_h\varphi_h(0)| = |\varphi_h \odot \mathcal{X}_h(0)| = \left|\int\limits_{\mathbb{T}} \mathcal{X}_h(t)\varphi_h(t)\,dt\right| = E_{n-1}(\mathcal{X}_h)_L = E_{n-1}(\mathcal{X}_h)_L \|\varphi_h\|_{L_\infty}.$$

Thus, inequality (1.3) is sharp; i.e.,

$$c(h,n) := \sup_{\varphi\in\mathcal{T}_{n-1}^{\perp},\,\varphi\not\equiv 0} \frac{\|\varphi * \mathcal{X}_{(-h,h)}\|_C}{\|\varphi\|_{L_\infty}} = \sup_{\varphi\in\mathcal{T}_{n-1}^{\perp},\,\varphi\not\equiv 0} \frac{\|\varphi \odot \mathcal{X}_h\|_C}{\|\varphi\|_{L_\infty}} = E_{n-1}(\mathcal{X}_h)_L \tag{1.4}$$

for any $n \in \mathbb{N}$ and $h>0$. From this and (0.8), we find that

$$c(h,n) \le \frac{\pi}{2nh} < 1 \quad \text{for} \quad h > \frac{\pi}{2n}. \tag{1.5}$$

We denote by W_2 the operator that assigns to a function $f \in C$ and a number $h>0$ the following function (see [1, Ch. 5, Sect. 83]):

$$W_2(f,h,x) := f(x) - S_h f(x) = f(x) - f * \mathcal{X}_{(-h,h)}(x) = f(x) - f{\odot}\mathcal{X}_h(x). \tag{1.6}$$

This operator can be written in the form

$$W_2(f,h,x) = -\frac{1}{2h}\int\limits_0^h \Big(f(x-t) - 2f(x) + f(x+t)\Big)\,dt.$$

To an element f from the space C, we assign the following function of variable $h > 0$:

$$W_2(f,h) := \|f - S_h f\|_C = \sup_{x\in\mathbb{R}} |W_2(f,h,x)|.$$

It is clear that

$$2W_2(f,h) \le \frac{1}{h}\int\limits_0^h \omega_2(f,t)\,dt \le \omega_2(f,h), \tag{1.7}$$

where

$$\omega_2(f,h) := \sup_{x\in\mathbb{R},\,0\le t\le h} |f(x-t) - 2f(x) + f(x+t)|$$

is the *modulus of continuity of the second order of the function* f.

2. RELATION BETWEEN THE INTEGRAL APPROXIMATION OF THE FUNCTION $\mathcal{X}_h$ AND THE JACKSON INEQUALITY IN C FOR THE CASE OF THE SECOND MODULUS OF CONTINUITY

In this section, we present an application of the results about the integral approximation of the function $\mathcal{X}_h$ by the subspace $\mathcal{T}_{n-1}$ (see the Introduction) for the investigation of the Jackson inequality between the value

$$E_{n-1}(f) = \min\{\|f-g\|_C :\ g \in \mathcal{T}_{n-1}\}$$

of the best uniform approximation of a function $f \in C$ and its second modulus of continuity.

The problem about constants in the Jackson and Jackson–Stechkin inequalities with the modulus of continuity of an arbitrary order has a rich history, which is partially described in monographs [1, 4–6] and papers [8, 10]. Below we prove the following statement.

Theorem. *Let $f \in C$, $n \in \mathbb{N}$, and $h > \pi/(2n)$. Then, the following inequalities hold:*

$$E_{n-1}(f) \le \frac{W_2(f,h)}{1-c(h,n)} \le \frac{1}{2h(1-c(h,n))}\int\limits_0^h \omega_2(f,t)\,dt \le \frac{\omega_2(f,h)}{2(1-c(h,n))}. \tag{2.1}$$

As a consequence, the following inequalities hold:

$$E_{n-1}(f) \le \frac{W_2(f,h)}{1-\pi/(2nh)} \le \frac{\omega_2(f,h)}{2-\pi/(nh)}. \tag{2.2}$$

To prove the theorem, we need the following lemma, where we use the standard notation

$$\widehat{F}_k := \frac{1}{2\pi}\int\limits_{-\pi}^{\pi} F(t)e^{-ikt}\,dt, \quad k \in \mathbb{Z},$$

for the Fourier coefficients of a function $F \in L$.

Lemma. *Let $F \in L$ and $2\pi\widehat{F}_k \neq 1$ for all $k \in \mathbb{Z}$. Then, for arbitrary $n \in \mathbb{N}$ and $g \in \mathcal{T}_{n-1}$, a polynomial $\tau \in \mathcal{T}_{n-1}$ exists such that*

$$g = \tau - \tau \odot F. \tag{2.3}$$

Proof. As is known (see [9, Ch. 3, Sect. 3.1]), the convolution $\varphi \odot \psi$ (see (1.1)) of functions $\varphi, \psi \in L$ belongs to L and the following formula is valid for the Fourier coefficients of the convolution:

$$\widehat{(\varphi \odot \psi)}_k = 2\pi \widehat{\varphi}_k \widehat{\psi}_k \quad \text{for all} \quad k \in \mathbb{Z}. \tag{2.4}$$

We find from (2.3) and (2.4) that

$$\widehat{g}_k = \widehat{\tau}_k \left(1 - 2\pi \widehat{F}_k\right), \quad \widehat{\tau}_k = \frac{\widehat{g}_k}{1 - 2\pi \widehat{F}_k} \quad \text{for} \quad |k| \leq n-1.$$

Thus, the required polynomial τ has the form

$$\tau(x) = \sum_{|k| \leq n-1} \frac{\widehat{g}_k e^{ikx}}{1 - 2\pi \widehat{F}_k}. \tag{2.5}$$

The lemma is proved. □

Proof of the theorem. We denote by $g_h \in \mathcal{T}_{n-1}$ the polynomial of the best integral approximation of the function $\mathcal{X}_h$.

By the lemma, for an arbitrary function $f \in C$, a trigonometric polynomial $\tau_f \in \mathcal{T}_{n-1}$ exists such that the following equality holds:

$$f - \tau_f = (f - \tau_f) \odot (\mathcal{X}_h - g_h) + W_2(f, h, \cdot). \tag{2.6}$$

Indeed, by (1.6), equation (2.6) is equivalent to each of the following equations:

$$f - \tau_f = (f - \tau_f) \odot (\mathcal{X}_h - g_h) + f - f \odot \mathcal{X}_h,$$

$$-\tau_f = (f - \tau_f) \odot (\mathcal{X}_h - g_h) - f \odot \mathcal{X}_h,$$

$$f \odot g_h = \tau_f - \tau_f \odot (\mathcal{X}_h - g_h).$$

The last of these equations has form (2.3), where

$$g = f \odot g_h, \qquad \tau = \tau_f, \qquad F = \mathcal{X}_h - g_h. \tag{2.7}$$

Under the assumptions of the theorem, $h > \pi/(2n)$; therefore, in view of (0.8), we have $\|F\|_L < 1$. Consequently, $|2\pi \widehat{F}_k| < 1$ for all $k \in \mathbb{Z}$. Thus, the assumptions of the lemma are satisfied; so, equation (2.6) has a solution. Hence, we find

$$\|f - \tau_f\|_C \leq \|f - \tau_f\|_C E_{n-1}(\mathcal{X}_h)_L + W_2(f, h).$$

Taking into account (1.4), we arrive at the inequality

$$\|f - \tau_f\|_C \leq \frac{W_2(f, h)}{1 - c(n, h)}. \tag{2.8}$$

Inequalities (2.1) follow from (1.7) and (2.8). In view of (1.5) and (1.7), (2.1) imply (2.2). □

Remark 1. Analyzing the proof of the theorem (see (2.4), (2.5), and (2.7)) and taking into account (0.2), we conclude that the formula

$$\tau_f(x) = \sum_{|k|\le n-1} \frac{\widehat{g}_k\, e^{ikx}}{1-2\pi\widehat{F}_k} = \sum_{|k|\le n-1} \frac{2\pi\,\widehat{g}_k(h)\,\widehat{f}_k\, e^{ikx}}{1+2\pi\widehat{g}_k(h)-\dfrac{\sin kh}{kh}} \tag{2.9}$$

(where $(\sin kh)/(kh) = 1$ for $k = 0$ and $\widehat{g}_k(h)$ is the kth Fourier coefficient of the polynomial g_h) provides the approximation method mentioned above, i.e., the method that assigns to a function $f \in C$ trigonometric polynomial (2.9) least deviating from f with error (2.8).

Remark 2. The following inequality is known:

$$E_{n-1}(f)_C \le \omega_2\left(f, \frac{\pi}{2n}\right), \quad f \in C, \quad n \in \mathbb{N}, \tag{2.10}$$

which is sharp in the following sence:

$$\mathcal{K}(\pi/2) = 1,$$

where

$$\mathcal{K}(\delta) := \sup_{n\in\mathbb{N}} \sup_{f\in C,\, f\not\equiv\mathrm{const}} \frac{E_{n-1}(f)_C}{\omega_2(f,\delta/n)}, \quad \delta > 0.$$

Zhuk and Shalaev obtained the upper and lower estimates for the value $\mathcal{K}(\pi/2)$, respectively (see [5, Ch. 8, Sect. 3, Theorem 3 and Comments to Ch. 8, Sect. 3]).

Remark 3. Though the first inequality in (2.1) is not sharp, a modification of the suggested approach intended for the proof of direct theorems of approximation theory allows us to obtain the sharp inequality

$$E_{n-1}(f)_C \le \bigl(\tan(1) + \sec(1)\bigr)\, W_2\Bigl(f, \frac{\pi}{2n}\Bigr), \tag{2.11}$$

which gives the classical Favard–Akhiezer–Krein inequality (in the case of the second derivative) with a small loss in the constant; namely,

$$E_{n-1}(f)_C \le \bigl(\tan(1) + \sec(1)\bigr)\, W_2\Bigl(f, \frac{\pi}{2n}\Bigr) \le \frac{\tan(1)+\sec(1)}{3}\,\frac{3n}{\pi}\int\limits_0^{\pi/(2n)} t^2 |f^{(2)}(t)|\,dt;$$

therefore,

$$E_{n-1}(f)_C \le \lambda\,\frac{\pi^2}{8n^2}\,\|f^{(2)}\|_{L_\infty}, \quad \text{where} \quad \lambda = \frac{\tan(1)+\sec(1)}{3} = 1.1360744807\ldots. \tag{2.12}$$

The constant in inequality (2.12) is factor of λ greater than the sharp constant in the following Favard–Akhiezer–Krein inequality (see [6, Ch. 4, Sect. 4.2, Theorem 4.2.1, formula (2.5); Ch. 3, Sect. 3.1, formulas (1.8) and (1.9); Comments to Ch. 4, Sects. 4.1, 4.2]):

$$E_{n-1}(f)_C \le \frac{\pi^2}{8n^2}\,\|f^{(2)}\|_{L_\infty}. \tag{2.13}$$

ACKNOWLEDGMENTS

The first author was supported by the Russian Foundation for Basic Research (project no. 08-01-00213), by the Integration Project for Fundamental Research of the Ural and Siberian Divisions of the Russian Academy of Sciences, and by the Program for State Support of Leading Scientific Schools of the Russian Federation (project no. NSh-1071.2008.1).

The second author was supported by the Government of Poland (project no. 201 016 31/1206).

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Translated by M. Deikalova